奧冠教育中心

OLYMPIAD CHAMPION EDUCATION CENTRE

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泰國國際數學競賽初賽 2020 - 2021

THAILAND INTERNATIONAL MATHEMATICAL OLYMPIAD HEAT ROUND 2020 - 2021

高中組 Senior Secondary Group

時限：90 分鐘

Time allowed: 90 minutes

試題

Question Paper

考生須知：

Instructions to Contestants:

1. 本卷包括 試題 乙份，試題紙不可取走。
   Each contestant should have ONE Question-Answer Book which CANNOT be taken away.
2. 本卷共 5 個範疇，每範疇有 5 題，共 25 題，每題 4 分，總分 100 分，答錯不扣分。
   There are 5 exam areas and 5 questions in each exam area. There are a total of 25 questions in this Question-Answer Book. Each carries 4 marks. Total score is 100 marks. No points are deducted for

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.

請將答案寫在 答題紙 上。

All answers should be written on the ANSWER SHEET.

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incorrect answers.

3. 請將答案寫在 答題紙 上。
   All answers should be written on ANSWER SHEET.
4. 比賽期間，不得使用計算工具。
   NO calculators can be used during the contest.
5. 本卷中所有圖形不一定依比例繪成。
   All figures in the paper are not necessarily drawn to scale.
6. 比賽完畢時，本試題會被收回。
   This Question-Answer Book will be collected at the end of the contest.

本試題不可取走。

THIS Question-Answer Book CANNOT BE TAKEN AWAY.

未得監考官同意，切勿翻閱試題，否則參賽者將有可能被取消資格。

DO NOT turn over this Question-Answer Book without approval of the examiner.

Otherwise, contestant may be DISQUALIFIED.

請將答案寫在 答題紙 上。

All answers should be written on the ANSWER SHEET.

Open-Ended Questions (1^{st} ~25^{th}) (4 points for correct answer, no penalty point for wrong answer)

Logical Thinking

1. Charlie goes east for 26km, then goes south for 42km, goes north for 6km and goes west for 11km. How far is he now from the original position?

   Charlie pergi ke Timur sejauh 26km, kemudian pergi ke Selatan sejauh 42km, ke Utara sejauh 6km dan ke Barat sejauh 11km. Berapa jauh dia sekarang dari posisi awalnya?

2. A box contains 2020 balls which is either Red, Green, Blue, Yellow, White or Black. If 1920 balls are drawn, we can ensure that at least one ball of each colour is drawn. Find the minimum number of the balls needed to be drawn to ensure that balls of 4 different colors are taken.

   Sebuah kotak berisi 2020 bola berwarna merah, hijau, biru, kuning, putih dan hitam. Jika 1920 bola diambil, kita dapat memastikan setidak satu dari masing-masing warna bola terambil. Carilah berapa banyak bola harus diambil untuk memastikan ada 4 warna bola yang terambil.

3. 17^{th} February, 2020 is Monday, which date of the week will 19^{th} January, 2028 be?

   17 Februari 2020 adalah hari Senin, hari apakah tanggal 19 Januari 2028?

4. There are 38 candies, Player *A* and *B* take candy alternately, starting from *A*, each player can take 1 to 4 candy(ies) and then pass to the other player. If the player who takes the last candy wins the game, and *A* takes 2 candies first, how many candy(ies) should Player *B* take in his first move to ensure he will win the game?

   Terdapat 38 permen, Pemain A dan B mengambil permen bergantian, mulai dari A, masing-masing pemain mengambil 1 sampai 4 permen dan memberikan kepada pemain satunya. Jika pemain yang mengambil permen terakhir menang, dan A mengambil 2 permen pertama-tama, berapa banyak permen yang harus diambil Pemain B untuk memastikan dia akan menang?

5. Given x and y are prime numbers and $x^2 - 3y^3 = 1$, find the value of $x + y$.

   Diketahui x dan y adalah bilangan prima dan $x^2 - 3y^3 = 1$, carilah nilai dari $x + y$.

請將答案寫在 答題紙 上。

All answers should be written on the ANSWER SHEET.

Algebra

6. Solve the equation $\log_6\left(x^2+11\right)+\log_6 18-2\log_6 x=3$.

   Pecahkan persamaan $\log_6\left(x^2+11\right)+\log_6 18-2\log_6 x=3$.

7. It is known that $2^{2020}=k$. Find the remainder of $\left(x^{2020}-10x+18\right)\div\left(x^2-4\right)$. (Answer in term of k)

   Diketahui $2^{2020}=k$. Carilah sisa dari $\left(x^{2020}-10x+18\right)\div\left(x^2-4\right)$. (Jawablah dalam bentuk k)

8. Find the coefficient of x^3 in the expansion of $\left(1+3x-x^2-x^3\right)^3$.

   Carilah koefisien dari x^3 pada pengembangan dari $\left(1+3x-x^2-x^3\right)^3$.

9. Find the value of x such that $x=\sqrt{x}+30$.

   Carilah nilai dari x agar $x=\sqrt{x}+30$.

10. Factorize $2x^2+5xy-3y^2+14y-8$.

    Faktorisasikan $2x^2+5xy-3y^2+14y-8$.

Number Theory

11. Given that $\overline{168A248BC}$ is a 9-digit number which is divisible by 66, find the maximum value of $A\times B\times C$.

    Diketahui $\overline{168A248BC}$ adalah bilangan 9-angka yang dapat diabgi 66, carilah nilai maksimum dari $A\times B\times C$.

12. Find the remainder when 222^{2020} is divided by 16.

    Carilah sisa dari 222^{2020} dibagi 16.

請將答案寫在 答題紙 上。

All answers should be written on the ANSWER SHEET.

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13. There are 3 distinct prime numbers. Their product is 17 times of their sum. Find the greatest prime number among them.

    Terdapat 3 bilangan prima berbeda. Hasil perkalian dari ketiga bilangan prima ini tiga kali lebih besar dari hasil penjumlahnnya. Carilah bilangan prima yang terbesar di antara ketiganya.

14. Find the maximum value of the integer which is smaller than $\left(2+\sqrt{3}\right)^4$.

    Carilah nilai maksimum dari bilangan bulat yang lebih kecil dari $\left(2+\sqrt{3}\right)^4$.

15. Express $\left(\cos\frac{\pi}{1010}+i\sin\frac{\pi}{1010}\right)^{2020}$ in the form of $a+bi$.

    Nyatakan $\left(\cos\frac{\pi}{1010}+i\sin\frac{\pi}{1010}\right)^{2020}$ dalam bentuk $a+bi$.

Geometry

16. Find the area enclosed by the y-axis and the straight lines $2x=y+8$ and $3y=9-5x$.
    Carilah luas dari area yang dibatasi oleh sumbu-y dan garis lurus $2x=y+8$ dan $3y=9-5x$.

17. Given that $\tan x=\frac{\sqrt{5}}{2}$. Find the value of $\cos 2x+\sin 2x$.

    Diketahui $\tan x=\frac{\sqrt{5}}{2}$. Carilah nilai dari $\cos 2x+\sin 2x$.

請將答案寫在 答題紙 上。

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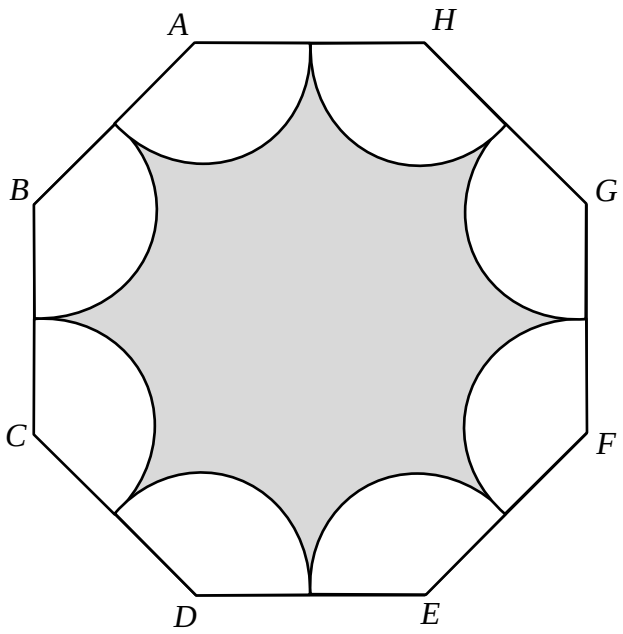
18. The figure below is formed by a regular-octagon and 8 identical sectors. If the side length of the octagon is 2 and the radii of the sectors are 1, find the area of shadowed area. (Answer in term of π and surd form)

    Gambar di bawah ini dibentuk dari sebuah segidelapan beraturan dan 8 sektor lingkaran. Jika panjang sisi segidelapan adalah 2 dan jari-jari sektor lingkaran adalah 1, carilah luas dari area yang diarsir. (Jawablah dalam bentuk π dan akar)



Question 18

Soal no. 18

19. Find the sum of slope of the two angle bisectors of $5x+y-2=0$ and $x-2y-1=0$.
    Carilah hasil penjumlahan dari dua garis pembagi sudut $5x+y-2=0$ dan $x-2y-1=0$.

請將答案寫在 答題紙 上。

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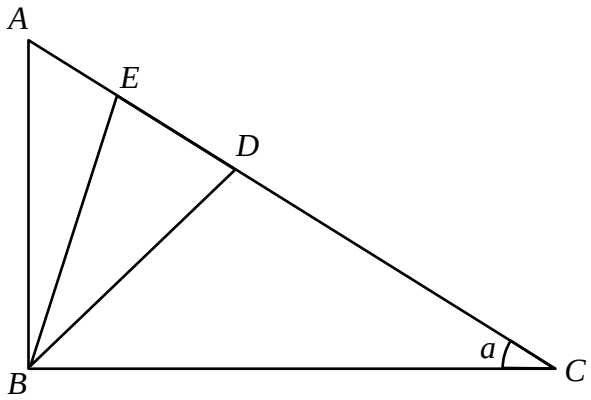
20. In the figure below, ABC is a right-angled triangle. BED is an isosceles triangle and $BD = BE$. If $BD = DC$ and $\angle BCD = a$, find the area ratio of ΔBED and ΔABC. (Answer in term of a)

Pada gambar di bawah ini, ABC adalah sebuah segitiga siku-siku. BED adalah sebuah segitiga samakaki dan $BD = BE$. Jika $BD = DC$ dan $\angle BCD = a$, carilah rasio dari luas ΔBED dan ΔABC. (Jawablah dalam bentuk a)



Question 20

Soal no. 20

Combinatorics

21. 5 boys and 4 girls sit on their own chairs. Now they stand up and choose a chair to sit randomly. How many possible cases are there that "only one boys and three girls not sit on their own chair?"
5 anak laki-laki dan 4 anak perempuan duduk di kursi mereka masing-masing. Sekarang mereka berdiri dan memilih sebuah kursi secara acak. Ada berapa kemungkinan yang mungkin untuk terjadi "hanya satu anak laki-laki dan tiga anak perempuan tidak duduk pada kursi mereka"?

22. A fair 6-faced dice is thrown 5 times. Find the probability that the product of numbers obtained is 1-digit number.

Sebuah dadu bersisi-6 dilempar 5 kali. Carilah peluang hasil kali dari bilangan-bilangan yang didapatkan adalah bilangan 1-angka.

23. Find the number of all integral solutions of the equation of $7x + 6y = 2xy$.

Ada berapa banyak solusi bulat dari persamaan $7x + 6y = 2xy$?

請將答案寫在 答題紙 上。

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24. If $-2000 < x, y < 2000$ and x, y are integers. If $8x - 3y = 201$, find the maximum value of $x - y$.

    Jika $-2000 < x, y < 2000$ dan x, y adalah bilangan bulat. Jika $8x - 3y = 201$, carilah nilai maksimum dari $x - y$.

25. How many 3-digit positive integers whose sum of digit(s) is / are less than 13? (e.g. 151, 721)

    Berapa banyak bilangan bulat positif 3-angka yang hasil penjumlahan angka-angkanya kurang dari 13? (contoh 151, 721)

~ 全卷完 ~

~ End of Paper ~