International Junior Math Olympiad

GRADE 9

Time Allowed: 90 minutes

Name:

Country:

INSTRUCTIONS

1. Please DO NOT OPEN the contest booklet until told to do so.
2. There are 30 questions.

   Section A: Questions 1 to 10 score 2 points each, no points are deducted for unanswered question and 1 point is deducted for wrong answer.

   Section B: Questions 11 to 20 score 3 points each, no points are deducted for unanswered question and 1 point is deducted for wrong answer.

   Section C: Question 21 to 30 score 5 points each, no points are deducted for unanswered or wrong answer.
3. Shade your answers neatly using a 2B pencil in the Answer Entry Sheet.
4. No one may help any student in any way during the contest.
5. No electronic devices capable of storing and displaying visual information is allowed during the exam. Strictly NO CALCULATORS are allowed into the exam.
6. No exam papers and written notes can be taken out by any contestant.

SECTION A – 10 questions

Question 1

How many positive integers m are there such that $m^2 + 2017$ is a perfect square?

A. 1
B. 2
C. 3
D. 4
E. 6

Question 2

Let x satisfies the equation $\frac{1}{x} = \frac{1}{2017^2} + \frac{1}{2018^2} + \cdots + \frac{1}{4030^2}$. Which of the following numbers is the nearest to x?

A. 2016
B. 2017
C. 3024
D. 4035
E. 4037

Question 3

Triangles ABC and ADC are isosceles with AB = BC and AD = DC. Point D is inside triangle ABC, angle ABC measures 40 degrees, and angle ADC measures 140 degrees. What is the degree measure of angle BAD?

A. 20
B. 30
C. 40
D. 50
E. 60

Question 4

Real numbers *a* and *b* satisfy the equations $3^a = 81^{b+2}$ and $125^b = 5^{a-3}$. What is *ab*?

A. - 60
B. -17
C. 9
D. 12
E. 60

Question 5

Let x, y and z be real numbers such that $3x + y = 1$, $3y + z = \frac{1}{2}$, and $3z + x = -\frac{1}{2}$. What is the value of $x + y + z$?

A. 1
B. $\frac{1}{2}$
C. $\frac{1}{3}$
D. $\frac{1}{4}$
E. 0

Question 6

Which of the following is equal to $\frac{1+\sqrt{2}}{\sqrt{2}-1}$?

A. $1 + \sqrt{2}$
B. $3 + 2\sqrt{2}$
C. $3\sqrt{2}$
D. $2 + \sqrt{2}$
E. $1 + \frac{2}{3}\sqrt{2}$

Question 7

What are the last two digits of 2017^{2017}?

A. 77
B. 81
C. 93
D. 37
E. 57

Question 8

Students from Mrs. Hein's class are standing in a circle. They are evenly spaced and consecutively labelled using whole numbers starting from 1. The student in place number 3 is standing directly across the student in place number 17. How many students are there in Ms. Hein's class?

A. 28
B. 29
C. 30
D. 31
E. 32

Question 9

In triangle ABC, AC = 4, BC = 5, and 1 < AB < 9. Let D, E and F be the midpoints of BC, CA, and AB, respectively. AD and BE intersect at G and point G is on CF. How long is AB?

A. 2
B. 3
C. 4
D. 5
E. Not enough information

Question 10

A city is divided into four regions. The city council has decided that a new city hall, a new school, and a new movie theatre shall be built. The only condition is that the school and the movie theatre must not be in the same region. How many ways these four buildings be built in the city? *(Ignore the time of construction)*

A. 4

B. 16

C. 24

D. 48

E. 64

Section B – 10 questions

Question 11

Three points A, B, and C have coordinates (0, 4), (6, 2), and (10, 4), respectively. Then ∠ABC equals ______.

A. 105°

B. 120°

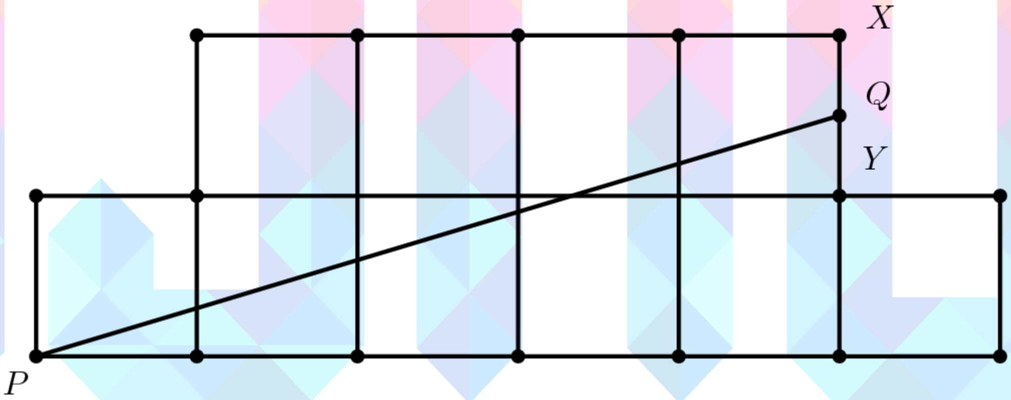
C. 135°

D. 145°

E. None of the above

Question 12

The diagram shows an octagon consisting of 10 unit squares. The shapes below *PQ* is a unit square and a triangle with base 5. If PQ divides the area of the octagon into two equal parts, what is the value of $\frac{XQ}{QY}$?



A. $\frac{2}{5}$

B. $\frac{1}{2}$

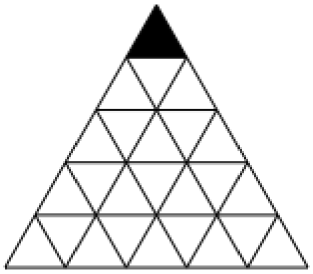
C. $\frac{3}{5}$

D. $\frac{2}{3}$

E. $\frac{3}{4}$

Question 13

In the figure, two triangles are considered neighbours if they have a side or a point in common. You can only move from one triangle to its neighbouring triangle. How many possible shortest paths are there to the bottom row from the black triangle?



A. 81
B. 153
C. 215
D. 375
E. 945

Question 14

Suppose *a* satisfies the equation $4 = a + a^{-1}$. What is the value of $a^4 + a^{-4}$?

A. 164
B. 172
C. 192
D. 194
E. 212

Question 15

Which one of these numbers must be placed in the middle (3rd) if they are to be arranged in increasing or decreasing order?

A. π
B. $\sqrt{12}$
C. $\frac{7}{2}$
D. $\frac{\sqrt{11}+\sqrt{13}}{2}$
E. $\frac{2}{\frac{1}{\sqrt{11}}+\frac{1}{\sqrt{13}}}$

Question 16

The numbers a_1, a_2, a_3 and a_4 are drawn one at a time from the set {0, 1, 2, …, 9}. If these four numbers are drawn with replacement, what is the probability that $a_1 a_4 - a_2 a_3$ is an even number?

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. $\frac{3}{8}$

D. $\frac{3}{4}$

E. $\frac{5}{8}$

Question 17

Per, Ragnar, and Lars live in the same neighbourhood. They have found out that the straight line distance from Per's house to Ragnar's house is 250 m, and from Ragnar's house to Lars' house is 300 m. Which of the following is true about the distance between Per's house and Lars' house?

A. The distance is precisely 550 m.

B. The distance is between 0 m and 550 m.

C. The distance is between 50 m and 550 m.

D. The distance is between 250 m and 300 m.

E. The distance can be any value.

Question 18

You throw three regular six-sided dice. What is the probability that you get one odd number and two even numbers?

A. $\frac{1}{4}$

B. $\frac{3}{8}$

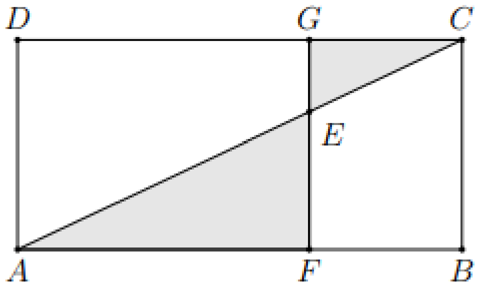
C. $\frac{4}{27}$

D. $\frac{1}{2}$

E. $\frac{1}{3}$

Question 19

The area of the rectangle ABCD is 1. Assume that E is on the diagonal AC, and that the line through E parallel to AD meets AB at F and CD at G. Let $x = \frac{AE}{EC}$. What is the sum of the areas of AFE and ECG in terms of x?



A. $\frac{1+x^2}{2(1+x)^2}$

B. $\frac{2(1+x^2)}{(1+x)^2}$

C. $\frac{2(1+x)^2}{1+x^2}$

D. $\frac{(1+x)^2}{2(1+x^2)}$

E. Not enough information

Question 20

All numbers can be written in base three, in a way similar to base ten. The difference is that in base three, only the digits 0, 1, and 2 can be used. The numbers which we write in base ten as 1, 2, 3, 4, 5, and so on, are written in base three as 1, 2, 10, 11, 12, and so forth. What number in base ten corresponds to the number 1021 in base three?

A. 16

B. 31

C. 34

D. 40

E. 51

Section C – 10 questions

Question 21

The plane left city A at 08:00 and arrived in city B at 10:00. Then it left city of B at 12:00 and arrived in city A at 18:00. If the duration of the flight in both directions is the same, what is the duration of the flight in minutes?

Question 22

Consider the sequence of real numbers (a_n), for n = 1, 2, 3, …,

$a_1 = 2$ and $a_n = (\frac{n+1}{n-1})(a_1 + \cdots + a_{n-1})$ for all n ≥ 2.

Determine the value of $\frac{a_{2017}}{2^{2017}}$.

Question 23

One of the famous Hungarian mathematicians lived all his life in the 19th century (1801-1900). Three of the digits in his year of birth and his year of death are the same. His birth year is a multiple of 17, and his year of death is a multiple of 31. If he lived for more than 50 years, what year was he born?

Question 24

Let $p(x) = x^4 + ax^3 + bx^2 + cx + d$, where a, b, c, d are real numbers. It is known that $p(1) = 841$, $p(2) = 1682$ and $p(3) = 523$. Find $\frac{p(9)+p(-5)-2}{-8}$.

Question 25

Let us call a positive integer "lucky" if its digits can be divided into two groups so that the sum of the digits in each group is the same. For example, 34175 is lucky because $3 + 7 = 4 + 1 + 5$. Find the smallest 4-digit lucky number, whose neighbor is also a lucky number (i.e. the integer next to it is a lucky number as well).

Question 26

For each positive integer x, let S(x) denotes the sum of its digits. Find the smallest positive integer n such that $9S(n) = 16[S(2n)]$.

Question 27

A box contains a total of 400 tickets of five different colours: blue, green, red, yellow and orange. The ratio of blue to green to red tickets is 1: 2: 4. The ratio of green to yellow to orange tickets is 1: 3: 6. What is the smallest number of tickets that must be drawn to ensure that at least 50 tickets of one colour have been selected?

Question 28

There are eight positive integers in a row. Starting from the third, each is the sum of the two numbers before it. If the eighth number is 2017, what is the largest possible value of the first one?

Question 29

There are 10 children in a row. In the beginning, the total number of marbles girls have were equal to the total number of marbles boys have. Then each child gave a marble to every child standing to the right of him (or her). After that, the total number of marbles girls have increased by 25. How many girls are there in the row?

Question 30

Anna and Birger walks at the same constant speed. They start at the same place, facing in the same direction. Birger walked in that direction throughout the time. Anna, however, turned 90 degrees to the right immediately after her first step. Then she turned another 90 degrees to the right immediately after taking two more steps, and yet another 90-degree turn to the right immediately after taking four more steps. She walked this way, doubling the number of steps every time she turns. When Anna was about to turn for the 9th time, both of them stopped.

Find the integer part of the distance between them when they stopped.

END OF PAPER

1	A
2	D
3	D
4	E
5	D
6	B
7	A
8	A
9	E
10	D
11	C
12	D
13	B
14	D
15	D
16	A
17	C
18	B
19	A
20	C
21	0240
22	1009
23	1802
24	5621
25	1449
26	0011
27	0196
28	0010
29	0005
30	0319