International Junior Math Olympiad

GRADE 8

Time Allowed: 90 minutes

Name:

Country:

INSTRUCTIONS

1. Please DO NOT OPEN the contest booklet until told to do so.
2. There are 30 questions.

   Section A: Questions 1 to 10 score 2 points each, no points are deducted for unanswered question and 1 point is deducted for wrong answer.

   Section B: Questions 11 to 20 score 3 points each, no points are deducted for unanswered question and 1 point is deducted for wrong answer.

   Section C: Question 21 to 30 score 5 points each, no points are deducted for unanswered or wrong answer.
3. Shade your answers neatly using a 2B pencil in the Answer Entry Sheet.
4. No one may help any student in any way during the contest.
5. No electronic devices capable of storing and displaying visual information is allowed during the exam. Strictly NO CALCULATORS are allowed into the exam.
6. No exam papers and written notes can be taken out by any contestant.

SECTION A – 10 questions

Question 1

If $a \oplus b = \frac{a \times b}{a+b}$ for positive integers a and b, then what is $5 \oplus 10$?

A. $\frac{3}{10}$
B. 1
C. 2
D. $\frac{10}{3}$
E. 50

Question 2

The difference between any two consecutive numbers in the list a, b, c, d, e is the same. If $b = 5.5$ and $e = 10$, what is the value of a?

A. 4.0
B. 4.5
C. 5.0
D. 5.5
E. None of the above

Question 3

What are the last two digits of 2017^{2017}?

A. 77
B. 81
C. 93
D. 37
E. 57

Question 4

Students from Mrs. Hein's class are standing in a circle. They are evenly spaced and consecutively numbered starting with 1. The student with number 3 is standing directly across from the student with number 17. How many students are there in Ms. Hein's class?

A. 28
B. 29
C. 30
D. 31
E. 32

Question 5

The following are the number of fishes that Tyler caught in nine outings last summer: 2, 0, 1, 3, 0, 3, 3, 1, 2. Which statement about the mean, median, and mode is true?

A. median < mean < mode
B. mean < mode < median
C. mean < median < mode
D. median < mode < mean
E. mode < median < mean

Question 6

In triangle ABC, $AC = 4$, $BC = 5$, and $1 < AB < 9$. Let D, E and F be the midpoints of BC, CA, and AB, respectively. If AD and BE intersect at G and point G is on CF, how long is AB?

A. 2
B. 3
C. 4
D. 5
E. Not enough information

Question 7

A city is divided into four regions. The city council has decided that a new city hall, a new school, and a new movie theatre shall be built. The only condition is that the school and the movie theatre must not be in the same region. How many ways these four buildings be built in the city? *(Ignore the time of construction)*

A. 4

B. 16

C. 24

D. 48

E. 64

Question 8

Anne and Beate together have \$120, Beate and Cecilie together have \$60, and Anne and Cecilie together have \$70. How much money do they have in total?

A. 120

B. 125

C. 130

D. 180

E. 190

Question 9

Which one of the following numbers is equal to $4^7 \times 2^4$?

A. 8^3

B. 8^6

C. 8^{11}

D. 8^{14}

E. 8^{28}

Question 10

Which one of the following numbers is equal to $\frac{2017^4-2016^4}{2017^2+2016^2}$?

A. 2016

B. 4031

C. 4033

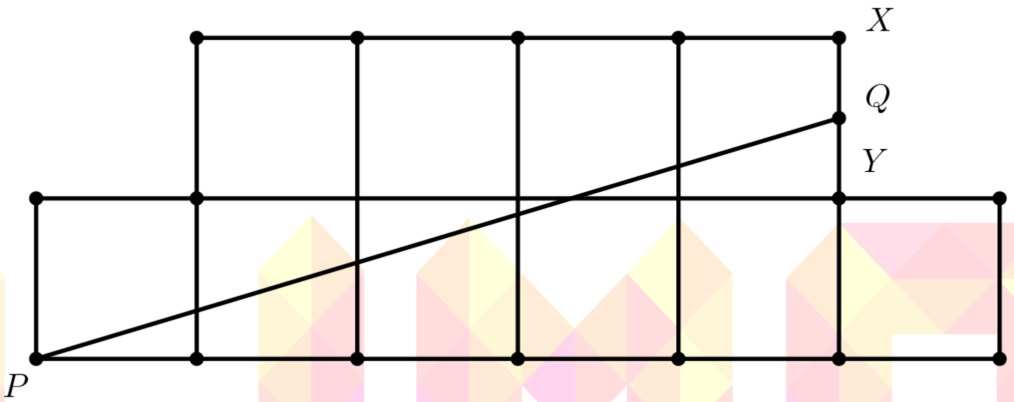
D. $2 \times (2017^2 - 2016^2)$

E. 2016×2017

Section B – 10 questions

Question 11

The diagram shows an octagon consisting of 10 unit squares. The shapes below *PQ* is a unit square and a triangle with base 5. If PQ divides the area of the octagon into two equal parts, what is the value of $\frac{XQ}{QY}$?



A. $\frac{2}{5}$

B. $\frac{1}{2}$

C. $\frac{3}{5}$

D. $\frac{2}{3}$

E. $\frac{3}{4}$

Question 12

If $a_1 + a_2 = 1$, $a_2 + a_3 = 2$, $a_3 + a_4 = 3$, $a_4 + a_5 = 4$, $\ldots a_{50} + a_{51} = 50$ and $a_{51} + a_1 = 51$, then what is the sum of $a_1, a_2, a_3, \ldots, a_{51}$?

A. 663

B. 1326

C. 1076

D. 538

E. 665

Question 13

The solution set of $\frac{x}{a}+\frac{1}{b}>0$ is $x<\frac{1}{3}$, where a and b are constants. Determine the solution set of $bx-a>0$.

A. $x>\frac{1}{3}$

B. $x<-\frac{1}{3}$

C. $x>-\frac{1}{3}$

D. $x<\frac{1}{3}$

E. None of the above

Question 14

A two-digit number formed by any 2 adjacent digits of a 2017-digit number is divisible by 17 or 23. If the last digit of the 2017-digit number is 1, find the first digit.

A. 2

B. 3

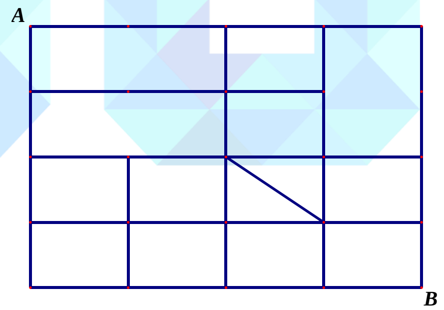
C. 4

D. 6

E. 9

Question 15

What is the number of shortest paths from A to B?



A. 4

B. 5

C. 6

D. 8

E. None of the above

Question 16

Which one of these numbers must be placed in the middle (3rd) if they are to be arranged in increasing or decreasing order?

A. π

B. $\sqrt{12}$

C. $\frac{7}{2}$

D. $\frac{\sqrt{11}+\sqrt{13}}{2}$

E. $\frac{2}{\frac{1}{\sqrt{11}}+\frac{1}{\sqrt{13}}}$

Question 17

The numbers a_1, a_2, a_3 , and a_4 are drawn one at a time from the set {0, 1, 2, …, 9}. If these four numbers are drawn with replacement, what is the probability that $a_1a_4 - a_2a_3$ is an even number?

A. $\frac{1}{2}$

B. $\frac{1}{4}$

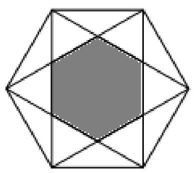
C. $\frac{3}{8}$

D. $\frac{3}{4}$

E. $\frac{5}{8}$

Question 18

There are two regular hexagons in the picture. What is the ratio of the area of the larger one to that of the smaller one?



A. 2:1

B. 3:1

C. $2\sqrt{3}$:1

D. 4:1

E. None of the above

Question 19

The sum of Anne's and Berit's ages is 60 years. Anne is three times as old as Berit was when Anne was the age that Berit is now. What is the sum of the digits of Anne's age?

A. 1
B. 3
C. 5
D. 7
E. 9

Question 20

Three points A, B, and C have coordinates (0, 4), (6, 2), and (10, 4), respectively. Then angle ∠ABC equals ______.

A. 105°
B. 120°
C. 135°
D. 145°
E. None of the above

Section C – 10 questions

Question 21

A series of bus tickets are labelled using all the numbers from 00000 through 99999. A girl collected all the tickets whose numbers are divisible by 78 and a boy collected all the tickets whose numbers are divisible by 77, but not by 78. How many more tickets did the girl collect?

Question 22

Six players compete in a tournament. Each player plays exactly two games against every other player. In each game, the winning player earns 2 points and the losing player earns 0 points. If the game results in a draw (tie), each player earns 1 point. What is the minimum possible number of points that a player needs to earn in order to guarantee that he/she will be champion (i.e he/she has more points than every other player)?

Question 23

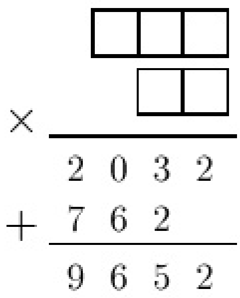
Let us call a positive integer "lucky" if its digits can be divided into two groups so that the sum of the digits in each group is the same. For example, 34175 is lucky because $3 + 7 = 4 + 1 + 5$. Find the smallest 4-digit lucky number, whose neighbor is also a lucky number (i.e. the integer next to it is a lucky number as well).

Question 24

For each positive integer n, define $S(n)$ to be the smallest positive integer divisible by each of the positive integers $1, 2, 3, \ldots, n$. For example, $S(5) = 60$. How many positive integers n are there such that $1 \leq n \leq 100$ and $S(n) = S(n + 4)$?

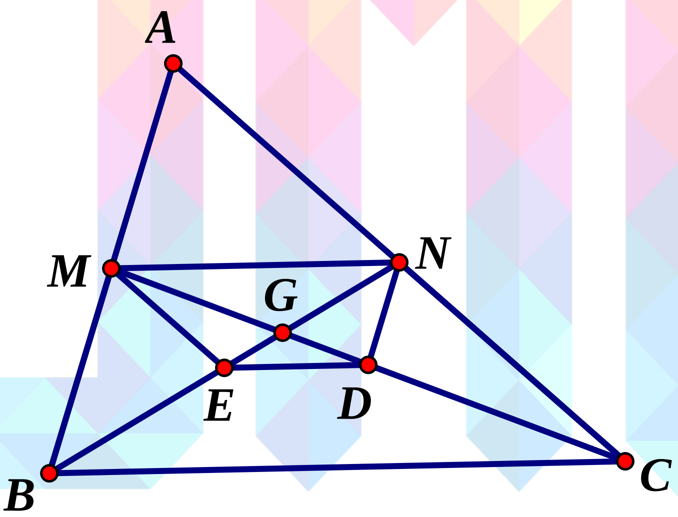
Question 25

Find the missing 3-digit number in the following multiplication.



Question 26

In triangle ABC, points M, N are the midpoints of AB, AC, respectively. Let D, E be the midpoints of CM, BN, respectively. Find the value of $\frac{Area\ of\ ABC}{Area\ of\ BCDE+Area\ of\ MNDE}$.



Question 27

One of the famous Hungarian mathematicians lived all his life in the 19th century (1801-1900). Three of the digits in his year of birth and his year of death are the same. His birth year is a multiple of 17, and his year of death is a multiple of 31. If he lived for more than 50 years, what year was he born?

Question 28

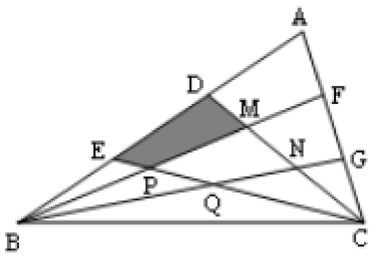
Let $p(x) = x^4 + ax^3 + bx^2 + cx + d$, where a, b, c, d are real numbers. It is known that $p(1) = 841$, $p(2) = 1682$ and $p(3) = 523$. Find the value of $\frac{p(9)+p(-5)-2}{-8}$.

Question 29

There are 10 children in a row. In the beginning, the total number of marbles girls have were equal to the total number of marbles boys have. Then each child gave a marble to every child standing to the right of him (or her). After that, the total number of marbles girls have increased by 25. How many girls are there in the row?

Question 30

As shown in the figure, the area of $\triangle ABC$ is 42. Points D and E divide the side AB into 3 equal parts, while F and G do the same thing to AC. CD intersects BF and BG at M and N, respectively. CE intersects BF and BG at P and Q, respectively. What is the area of the quadrilateral EPMD?



END OF PAPER

1	D
2	A
3	A
4	A
5	C
6	E
7	D
8	B
9	B
10	C
11	D
12	A
13	C
14	A
15	C
16	D
17	A
18	B
19	E
20	C
21	0001
22	0019
23	1449
24	0011
25	0254
26	0002
27	1802
28	5621
29	0005
30	0005